Week 3 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
   `23f3004032@ds.study.iitm.ac.in`
   or reach out on Discourse: `@23f3004032`

Quiz 1 May 22

1. What is the directional derivative of $f(x, y, z) = x^2y^3 - 4xz$ at $(1, 1, 1)$ in direction $[-1, 2, 0]$?

   [Ans: 3.3–3.7]

   For the matrix

   $$S = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 4 \end{bmatrix}$$

   what is the dimension of its row space?

   [Ans:3]

2. Consider system:

   $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

   Under what condition is it solvable?

   (a) $b_2 = 2b_1$

   (b) $b_2 = 3b_1$

   (c) $b_3 = 2b_1$

   (d) $b_3 = 3b_1$

   [Ans:c]

Quiz 2 May 22

ET May 22

Quiz 1 Sep 22

1. Two vectors v_1 and v_2 are such that $v_1 = 2v_2$. If $v_1 = (1, 2, 3)^T$ and $v_3 = (1, -5, 3)^T$, which statements are true?

   (a) v_1 and v_2 are parallel

   (b) v_1 and v_2 are perpendicular

   (c) v_2 and v_3 are not perpendicular

   (d) v_2 and v_3 are perpendicular

   [Ans: a, d]

2. What is the directional derivative of $f(x, y, z) = xyz$ at $(-1, 1, 3)$ along unit vector $\left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$?

   [Ans: 2.10–2.40]

3. For the matrix

$$S = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 4 \end{bmatrix}$$

   find the dimension of its column space.

   [Ans: 3]

Quiz 2 Sep 22

ET Sep 22

1. For a rectangle whose perimeter is 20 m, use the Lagrange multiplier method to find the dimensions that will maximize the area. What is the value of $\frac{2\times Length}{Breadth}$? [Ans: 2]

2. Which of the following statements is/are false?

   (a) A 3×3 matrix can have same column space and null space.

   (b) A 2×6 matrix can have same column space and null space.

   (c) Dimension of column space is always equal to the dimension of row space

   (d) Column space and row space of a matrix are the same.

   [Ans: 1, 2, 4]

3. What is the maximum possible nullity of the 4×4 orthogonal matrix? [Ans: 0]

Quiz 1 Jan 23

1. Which of the following statements is/are true?

   (a) There exists an $n \times n$ matrix A, n is odd, for which rank of A is not equal to nullity of A.

   (b) There exists an $n \times n$ matrix A, n is odd, for which rank of A is equal to nullity of A.

   (c) There exists an $n \times n$ matrix A, n is even, for which rank of A is not equal to nullity of A.

   (d) There exists an $n \times n$ matrix A, n is even, for which rank of A is equal to nullity of A.

   [Ans: a, c, d]

2. The direction of steepest descent for the function $f(x, y, z) = \cos(x)\sin(y)\cos(z)$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{4}\right)$ is:

   (a) $\left[-\frac{1}{4}, \frac{\sqrt{3}}{4}, -\frac{1}{4}\right]$

   (b) $\left[\frac{1}{4}, -\frac{\sqrt{3}}{4}, \frac{1}{4}\right]$

   (c) $\left[\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right]$

   (d) $\left[-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}\right]$

   [Ans: b]

3. Which of the following is the equation of the line that passes through point $[1, 2, 3, 4]$ and is tangent to the function

   $$f(p, q, r, s) = p^2 + q^2 + r^2 + s^2 + pq + rs - 2p - 2q$$

   at the point $[p, q, r, s] = [1, 3, 5, 1]$?

   (a) $[1, 2, 3, 4] + \alpha[3, 5, 11, 7]$

   (b) $[3, 5, 11, 7] + \alpha[1, 2, 3, 4]$

   (c) $[1, 2, 3, 4] + \alpha[0, 4, 10, 2]$

   (d) $[1, 2, 3, 4] + \alpha[5, 3, 11, 7]$

   (e) $[5, 3, 11, 7] + \alpha[1, 2, 3, 4]$

   [Ans: a]

4. If null space of a matrix A is span$\left\{ \begin{bmatrix} 7/17 \\ 22/17 \\ -1 \end{bmatrix} \right\}$, which of the following can be row space of A?

   (a) Span$\left\{ \begin{bmatrix} 1 \\ 0 \\ 7/17 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 22/17 \end{bmatrix} \right\}$

   (b) Span$\left\{ \begin{bmatrix} -1 \\ 0 \\ 7/17 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 22/17 \end{bmatrix} \right\}$

   (c) Span$\left\{ \begin{bmatrix} 1 \\ 0 \\ 7/17 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 22/17 \end{bmatrix} \right\}$

   [Ans: a]

5. Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 3 & 1 \\ 5 & 1 \end{bmatrix}$$

   which among the following is orthogonal to $C(A)$?

   (a) Span$\left\{ \begin{bmatrix} -3 \\ -5 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

   (b) Span$\left\{ \begin{bmatrix} 3 \\ 5 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

   (c) Span$\left\{ \begin{bmatrix} 3 \\ 5 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

   (d) Span$\left\{ \begin{bmatrix} -3 \\ -5 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

   [Ans: a]

6. For a 3×3 matrix A whose nullity is equal to 1, what is the value of

$$R(A) + \dim(C(A)) + \dim(C(A^T))?$$

Here $R(A)$ stands for rank of A, $\dim(C(A))$ stands for dimension of column space of A, and $\dim(C(A^T))$ stands for dimension of column space of A^T. Enter the answer as an integer.

[Ans: 6]

Quiz 2 Jan 23

ET Jan 23

1. If A is a 10×8 real matrix with rank 8, then which among the following is true?

   (a) There exists at least one $b \in \mathbb{R}^{10}$ for which the system $Ax = b$ has infinite number of solutions.

   (b) For every $b \in \mathbb{R}^{10}$, the system $Ax = b$ has infinite number of solutions.

   (c) There exists at least one $b \in \mathbb{R}^{10}$ for which the system $Ax = b$ has a unique solution.

   (d) For every $b \in \mathbb{R}^{10}$, the system $Ax = b$ has a unique solution.

   [Ans: 3]

Quiz 1 May 23

1. Which of the following statements is/are true?

   (a) Row space of a matrix is orthogonal to the null space of the matrix.

   (b) Row space of a matrix is orthogonal to the null space of transpose of the matrix.

   (c) Dimension of row space of a matrix is always equal to the dimension of column space of the matrix.

   (d) Dimension of row space of a matrix need not be equal to the dimension of column space of the matrix.

   [Ans: a, c]

2. Let the matrix $A \in \mathbb{R}^{d \times d}$ be diagonalizable, then which of the following statements is/are true for A?

   (a) A must be symmetric.

   (b) We can find a basis of eigenvectors for $\mathbb{R}^d$.

   (c) We can find an orthonormal basis of eigenvectors for $\mathbb{R}^d$.

   (d) A can have some repeated eigenvalues but must have distinct independent eigenvectors.

   [Ans: b, d]

3. Let $A, B \in \mathbb{R}^{n \times n}$ be two matrices, then which among the following statements are correct?

   (a) $x \in \text{Nullspace}(A) \Rightarrow x \in \text{Nullspace}(BA)$

   (b) $x \in \text{Columnspace}(A) \Rightarrow x \in \text{Columnspace}(B)$ where $A = BC$

   (c) $x \in \text{Nullspace}(AB) \Rightarrow Bx \in \text{Nullspace}(A)$

   (d) $x \in \text{Nullspace}(AB) \Rightarrow x \in \text{Nullspace}(B)$

   [Ans: a, b, c]

4. Let $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^2 + y^2$. Consider:

   $$X_{n+1} = X_n - h\nabla f(X_n), \quad h = 0.25, \quad X_0 = (1, 3)$$

   Which point does the algorithm converge to?

   (a) $(1, 3)$

   (b) $(1/2, 1/6)$

(c) $(0, 0)$

(d) The algorithm does not converge

[Ans: c]

5. Let P_1 be projection onto span$\{[1, 2, 3]^T\}$ and P_2 be the projection matrix that projects vectors onto the plane perpendicular to span of a. Find $\det(P_1 + P_2)$.

[Ans: 1]

6. Find directional derivative of

$$f(x, y, z) = x(x^2 - y^2) - z$$

at $(1, -1, 0)$ in direction $\left(\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$.

[Ans: -1.16 to -1.12]

7. **Comprehension:**

Let

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}.$$

Based on the above data, answer the given subquestions.

(a) Is the system $Ax = b$ consistent?

i. Yes

ii. No

[Ans: a]

(b) Using least square approximation, find length of projection of b onto column space of A.

[Ans: 3.12–3.16]

Quiz 2 May 23

ET May 23

1. Suppose an $m \times n$ matrix A satisfies $A^T A = I$ then which among the following is true?

   (a) Columns of A are orthonormal.

   (b) $Rank(A) = m$

   (c) $P = AA^T$ is a projection matrix.

   (d) $P = AA^T$ is not a projection matrix.

[Ans: 1, 3]

Quiz 1 Sep 23

1. Consider a vector $v = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Does v belong to the column space of

$$A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}?$$

   (a) Yes

   (b) No

[Ans: a]

2. At $P_0 = (1, 2)$, directional derivatives are given. Find derivative in direction $\left(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$.

[Ans: 4.88–4.95]

3. Let A be $n \times n$. Which statements are true?

   (a) If eigenvalues are $\lambda_1, \ldots, \lambda_n$, then A is similar to diagonal matrix

   (b) If $\text{rank}(A) = r$, then A has r non-zero eigenvalues

   (c) A may have complex eigenvalues

   (d) Repeated eigenvalue $\Rightarrow$ not diagonalizable

[Ans: c]

4. Select correct options.

   (a) There exists matrix whose row space contains $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and nullspace contains $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

   (b) There exists matrix with row space $\mathbb{R}^3$ and column space $\mathbb{R}^4$

   (c) There exists matrix with given column/row spaces

   (d) Nullspace defined by $x_1 + 2x_2 + 4x_3 = 0$

[Ans: c, d]

Quiz 2 Sep 23

ET Sep 23

1. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix}$. Find the nullspace of A.

   (a) $span\{(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix})\}$

   (b) $\{(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix})\}$

   (c) $span\{(\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix})\}$

   (d) $span\{(\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix})\}$

[Ans: 4]

Quiz 1 Jan 24

1. **Comprehension:**

   Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 2 & 5 & 6 \end{bmatrix}$$

   Based on this information, answer the given subquestions.

   (a) Find the row space of matrix A.

   i. $\text{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} \right\}$

   ii. $\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

   iii. $\text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

   [Ans: b]

   (b) Find the column space of matrix A.

   i. $\text{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} \right\}$

   ii. $\text{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

   iii. $\text{span}\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ -3 \\ 6 \end{bmatrix} \right\}$

   iv. $\text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \right\}$

   [Ans: a]

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a multivariable scalar-valued continuous function, such that

$$\frac{\partial f}{\partial x} = 2x + 1 \quad \text{and} \quad \frac{\partial f}{\partial y} = 2y + 1.$$

Suppose

$$\lim_{(x,y)\to(0,0)} f(x,y) = 5.$$

Find out the directional derivative of f at the point $(0,0)$ in the direction of the vector $(1,1)$.

(a) $\sqrt{2}$

(b) $2\sqrt{2}$

(c) $-2\sqrt{2}$

(d) $-\sqrt{2}$

[Ans: a]

Quiz 2 Jan 24

ET Jan 24

Quiz 1 May 24

1. Which of the following statements is true for a function $f(x, y)$ that is differentiable at a point (a, b)?

   (a) The directional derivative exists in every direction.

   (b) The directional derivative is the same in every direction.

   (c) The directional derivative can only be calculated along the axes.

   (d) The directional derivative does not exist.

   [Ans: a]

2. If $f(x, y) = x^2y + 3xy^2$, what is the length of the gradient at the point $(1, 2)$? (Enter the answer correct to two decimal places.) [Ans:20.58-20.64]

Quiz 2 May 24

ET May 24

1. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ Find the nullspace of A.

   (a) $span\{(\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix})\}$

   (b) $span\{(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix})\}$

   (c) $\{(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix})\}$

   (d) $span\{(\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}), (\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix})\}$

[Ans: 1]

Quiz 1 Sep 24

1. Which of the following 3×3 matrices have rank 2 with column space equal to row space?

   (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

   [Ans:a,d]

2. Consider the function $f(x, y) = x^2 + 3y^2$. In which direction does the maximum directional derivative occur at the point $(1, 2)$?

   (a) $\left(\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right)$

   (b) $\left(\frac{1}{\sqrt{37}}, \frac{6}{\sqrt{37}}\right)$

   (c) $\left(\frac{2}{\sqrt{5}}, \frac{24}{\sqrt{5}}\right)$

   (d) $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

   [Ans: b]

Quiz 2 Sep 24

ET Sep 24

Quiz 1 Jan 25

1. The directional derivative of a scalar function $f(x, y)$ at a point (x_0, y_0) in the direction of a unit vector u represents:

   (a) The rate of change of f in the direction of u at (x_0, y_0).

   (b) The magnitude of the gradient vector ∇f at (x_0, y_0).

   (c) The maximum value of the gradient at (x_0, y_0).

   (d) The angle between the vector u and ∇f.

   [Ans: a]

2. Consider a matrix
$$A = \begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}.$$
   Which of the following options is/are true?

   (a) Null space of A is span$\left\{ \begin{bmatrix} -5 \\ 2 \end{bmatrix} \right\}$

   (b) Column space of A is span$\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$

   (c) Left null space of A is span$\left\{ \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$

   (d) Row space of A is span$\left\{ \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\}$

   [Ans: a, b, c]

3. Consider a matrix $A_{20\times25}$. If the nullity of the matrix is 10, then find the rank of the matrix.

   [Ans: 15]

4. If $f(x, y, z) = \sin(xy) + e^{yz} + xz^2$, what is the length of the gradient at the point $(1, 0, 2)$? Enter the answer correct to two decimal places.

   [Ans: 6.37–6.43]

Quiz 2 Jan 25

ET Jan 25

Quiz 1 May 25

1. Consider a matrix
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}.$$
Which of the following options is/are true?

   (a) Null space of A is span$\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right\}$

   (b) Dimension of column space of A is 1

   (c) Left null space of A is span$\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$

   (d) Row space of A is span$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0.5 \end{bmatrix} \right\}$

[Ans: a, c, d]

2. Which of the following options is/are true regarding the projection matrix?

   (a) It is a symmetric matrix.

   (b) It is an orthogonal matrix.

   (c) It is orthogonally diagonalizable.

   (d) It is always a rank 1 matrix.

[Ans: a, c]

3. Consider a subspace $S = \{(x, y, z) \mid x + 2y + 2z = 0\}$ of $\mathbb{R}^3$. Which of the following options is/are true?

   (a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ is a vector of S.

   (b) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2.5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$ is an orthogonal basis of S.

   (c) The dimension of S is 1.

(d) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -2.5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$ is a basis of S.

[Ans: b, d]

Quiz 2 May 25

ET May 25

Quiz 1 Sep 25

1. Two vectors $v_1 = (1, 2, 2)^T$ and $v_2 = (2, -1, 0)^T$. Which of the following options is/are correct?

   (a) Vectors v_1 and v_2 are parallel to each other.

   (b) Vectors v_1 and v_2 are orthogonal to each other.

   (c) Projection of v_1 onto v_2 is the zero vector.

   (d) Projection of v_2 onto v_1 is the zero vector.

   [Ans: b, c, d]

2. Consider

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 5 \\ 2 & -1 & 1 \end{bmatrix}.$$

   (a) Which set forms a basis for row space of A?

   i. $\{(1, 0, 2), (0, 1, -1)\}$
   ii. $\{(1, 0, 2), (0, 1, -1), (0, 0, -1)\}$
   iii. $\{(1, 0, 2), (0, -1, 1), (0, -1, -4)\}$
   iv. $\{(0, 1, -1), (0, 0, -4)\}$

   [Ans: b]

   (b) Which linear relation is valid among rows?

   i. $R_2 - R_1 - R_3 = 0$
   ii. $R_1 + R_2 - 2R_3 = 0$
   iii. $0R_1 + 0R_2 + 0R_3 = 0$
   iv. $R_2 = 2R_1 + R_3$

   [Ans: c]

Quiz 2 Sep 25

ET Sep 25

1. For the system of equations: $x + y + z = 6$
   $2x + y - z = 1$
   $3x + 2y + z = 10$
   The solution (x, y, z) is:

   (a) (1, 2, 3)

   (b) (1, 3, 2)

   (c) (2, 1, 3)

   (d) (3, 2, 1)

[Ans: 1]